Economics 101b; Problem Set 2

Due September 12, 2001

1. Consider the production function:

$$\frac{Y}{L} = \left(\frac{K}{L}\right)^{\alpha} (E)^{1-\alpha}$$

a. Suppose α = 1/3, E=1, L=100, and K=64; what is output per worker Y/L?

The cube root of 64 is 4; the cube root of 100 is 4.64; so output per worker is 0.86

b. Suppose α = 1/3, E=3, L=196, and K=49; what is output per worker Y/L?

The cube root of 196 is 5.81, the cube root of 49 is 3.66, and 3 to the (2/3) power is 2.08; so output per worker is 1.31

c. If both capital K and labor L double, what happens to total output Y? (Not output per worker Y/L, but total output.)

Total output doubles too.

d. Holding E=1, suppose that capital per worker increases from 2 to 4 and then from 4 to 6. What happens to output per worker?

Output per worker increases from 1.26 to 1.59, and then from 1.59 to 1.82—jumping by 0.33 for the first additional dose of capital, and by 0.23 for the second.

2. Consider an economy in which the depreciation rate is 3% per year, the rate of population increase is 2% per year, the rate of technological progress is 1% per year, and the private savings rate is 19% of GDP. Suppose that the government increases its budget deficit--which had been at 1% of GDP for a long time--to 4% of GDP and keeps it there indefinitely.

a. What is the effect of this shift on the economy's steady-state capital-output ratio?

b. What is the effect of this shift on the economy's steady state growth path for output per worker?

c. Suppose that your forecast of output per worker 20 years in the future had been $100,000. What is your new forecast of output per worker twenty years hence?

a. If the increase in the budget deficit is not offset by an increase in the private saving rate or a capital inflow, this change in fiscal policy will reduce the national savings

rate from 18% of national income to 15% of national income. The steady-state capital-output ratio will fall from 3 to 2.5

b. It will lower the steady-state growth path—lower the level of output per worker on the steady state growth path at every point in time. The new steady-state growth path will have a level of output per worker equal to that on the old steady-state growth path times a factor:

(5/6)^λ

where the growth multiplier $\lambda = \alpha/(1-\alpha)$, where α is the diminishing returns to scale parameter in the production function.

c. It will be equal to \$100,000 x (5/6)^λ, where the growth multiplier $\lambda = \alpha/(1-\alpha)$, where α is the diminishing returns to scale parameter in the production function.

3. Consider an economy with the production function:

$$\frac{Y}{L} = \left(\frac{K}{L}\right)^{\alpha} (E)^{1-\alpha}$$

in which the depreciation rate on capital is three percent per year, the rate of population growth is one percent per year, and the rate of growth of labor-augmenting technology is one percent per year.

a. Suppose that the savings rate is ten percent of GDP. What is the steady-state capital-output ratio? What is the value of output per worker on the steady-state growth path written as a function of the level of labor-augmenting technology E and the diminishing-returns parameter α?

b. Suppose that the savings rate is fifteen percent of GDP. What is the steady-state capital-output ratio? What is the value of output per worker on the steady-state growth path written as a function of the level of labor-augmenting technology E and the diminishing-returns parameter α?

a. The steady-state capital-output ratio is 2. Along the steady-state growth path: $(Y_t/L_t) = 2^{\lambda} \times E_t$,
where the growth multiplier $\lambda = \alpha/(1-\alpha)$, where α is the diminishing returns to scale parameter in the production function.

b. The steady-state capital-output ratio is 3. Along the steady-state growth path:

$(Y_t/L_t) = 3^{\lambda} \times E_t$,

where the growth multiplier $\lambda = \alpha/(1-\alpha)$, where α is the diminishing returns to scale parameter in the production function.

4. What happens to the steady-state capital-output ratio if the rate of technological progress increases? Would the steady-state growth path of output per worker for the economy shift upward, downward, or remain in the same position?

The steady-state capital-output ratio falls. The steady-state growth path twists. For dates close in time to the date of the shift in technological progress, the new steady-state growth path is below the old one. For dates further in the future, the new steady-state growth path is above the old one.

Nevertheless, growth accelerates when technological progress increases: the boost to annual growth created by faster growth of the efficiency of labor outweighs the slowdown produced by the fact that the capital-output ratio is above its steady-state value, and is falling.

5. Discuss--that is, write two paragraphs evaluating--the following proposition: "An decrease in the savings rate will decrease the steady-state capital output ratio, and so decrease both output per worker and the rate of economic growth both in the first few years after the savings rate has increased and in the very long run as well."

An increase in the savings rate does increase the steady-state capital-output ratio, and does raise the level of output per worker along the steady-state growth path. In the first few years after the increase, it is indeed true that output per worker is above what it would otherwise have been, and that the growth rate of output per worker is above what it would otherwise have been as the economy converges to the new, higher steady-state growth path of output per worker.

In the very long run, after the economy has converged to its new steady-state growth path, output per worker is higher than it would have been. But the growth rate is not: remember that in the long run the growth rate of output per worker along the steady-state growth path does not depend on the savings rate, but only on the rate of technological progress g.

6. Would the steady-state growth path of output per worker for the economy shift upward, downward, or remain the same if capital were to become less durable--if the rate of depreciation on capital were to rise?

The steady-state growth path would shift downward. The steady-state capital-output ratio falls.

7. Suppose that a sudden burst of immigration increases a country's population and labor force, but does not affect its capital stock. Suppose further that the economy was on its steady-state growth path before the epidemic. What is the immediate effect of the immigration boom on output per worker? On the total economy-wide level of output? What happens subsequently? What does this logic suggest about the AFL-CIO's attitude toward immigration to the U.S. from Mexico?

It lowers output per worker, and raises the total economy-wide level of output. Thereafter the economy converges to the same steady-state growth path for output per worker as it had before (provided that the immigration wave is a once-and-for-all process, after which the rate of labor force growth returns to its old growth rate n). Such an immigration wave enriches owners of property (total output is higher, after all, and to a first approximation owners of property receive as income a constant share of total output). But such an immigration wave impoverishes workers: output per worker goes down, and it is a good bet that real wages will decline as well.

Hence the AFL-CIO's delicate attitude toward immigration. It sees immigrants (correctly) as a source of downward pressure on the wages of American workers. It also sees immigrants as working-class brothers and sisters who have come to America in search of a better life…