Economics 101b; Fall 2001; Problem Set 3

Due in class September 24, 2001

1. Suppose that the economy is well-described by the Solow growth model, with the diminishing-returns-to-capital parameter $\alpha = 1/3$, the depreciation rate $\delta = .04$ per year, the population growth rate n=.02 per year, and the rate of increase of the efficiency of labor g=.02 per year.

    a. Suppose that the national savings rate s = 0.32, 32%. What is the steady-state capital output ratio?

    b. Suppose that increased investment incentives and a large government budget surplus lower the savings rate s to 24%. What is the new steady-state capital output ratio?

    c. Suppose that in the year 2000 the efficiency of labor E is $10,000 a year. What is the level of GDP per worker in 2000 if the economy is on the steady-state growth path given in (a)? What is the level of GDP per worker in 2030 if the economy remains on the steady-state growth path given in (a)?

    d. How much, in percentage terms, is the steady-state growth path given in (b) below the steady-state growth path given in (a)?

    e. At approximately what rate will the capital output ratio converge towards its new steady state value in the year in which the savings rate is changed?

2. Suppose that the economy is well-described by the Solow growth model, with the diminishing-returns-to-capital parameter $\alpha = 1/4$, the depreciation rate $\delta = .04$, the population growth rate n=.02, and the rate of increase of the efficiency of labor g=.02.

    a. Suppose that the national savings rate s = 0.32, 32%. What is the steady-state capital-output ratio? If the efficiency of labor in year 2000 is equal to $20,000 a

year, what is the steady-state growth path level of output per worker in 2000? Knowing that total consumption C is equal to (1-s) x Y—that consumption is the amount of national product not devoted to savings and investment—what then is the steady-state growth path level of consumption per worker in 2000?

b. Suppose that the national savings rate s = 0.48, 48%. What is the steady-state capital-output ratio? If the efficiency of labor in year 2000 is equal to $20,000 a year, what is the steady-state growth path level of output per worker in 2000? What is the steady-state growth path level of consumption per worker in 2000?

c. Can you explain why your level of consumption per worker in case (b) was different from your level of consumption per worker in case (a)? Was it lower or higher

3. Suppose that population growth depends on the level of output per worker, so that:

(1) n = (.0001) x [(Y/L) - $200]

the population growth rate n is zero if output per worker equals $200, and that each $100 increase in output per worker raises the population growth rate by 1% per year.

Suppose also that the economy is in its *Malthusian* regime, so that the rate of increase of the efficiency of labor E is zero. Thus output per worker is given by:

(2) $$\frac{Y_t}{L_t} = \left(\frac{s}{n+\delta}\right)^{\left(\frac{\alpha}{1-\alpha}\right)} E_0$$

with the diminishing-returns-to-investment parameter $\alpha = .5$, with the depreciation rate $\delta = .04$, and with the efficiency of labor E_0 = $100.

a. Suppose that the savings rate s is equal to .08, 8% per year. Graph (on the same set of axes) steady-state output-per-worker (Y/L) as a function of the population growth rate n from equation (2) and the population growth rate n as a function of output-per-worker (Y/L) from equation (1).

b. Where do the curves cross? For what levels of output per worker Y/L and population growth n is the economy (i) on its steady-state path, and (ii) at its Malthusian rate of population growth?

c. Suppose that the savings rate were to rise by an infinitesimal amount--say by one-hundredth of one percentage point, from .08 to .0801. Calculate approximately how the equilibrium position of the economy would change. By how much--and in which direction--would steady-state output per worker change? By how much--and in which direction--would the population growth rate change?

4. Suppose that the economy is well described by the Solow growth model, with the diminishing-returns-to-investment parameter $\alpha = 1/2$, the depreciation rate $\delta = .03$, the population growth rate n=.01, and the rate of increase of the efficiency of labor g=.01. Suppose that the savings rate s = 20 and that in year 2000 the efficiency of labor E is $10,000.

a. What is the steady-state capital-output ratio?

b. What is the steady-state level of output per worker Y/L in 2000?

c. Suppose that actual output per worker in 2000 is $35,000. Is output per worker above or below its steady-state value?

d. Suppose that actual output per worker in 2000 is $35,000. Using the approximation that each year the economy closes a fraction:

$(1-\alpha) \times (n + g + \delta)$

of the gap between its current level of output per worker and its steady-state value of output per worker, calculate (approximately) what output per worker will be in 2001.

e. Suppose that actual output per worker in 2000 is $35,000. Using the same approximation, what (approximately) will output per worker be in 2010?

5. Many project that by the middle of the twenty-first century the population of the United States will be stable. Using the Solow growth model, what would such a downward shift in the growth rate of the labor force do to the growth of output per worker and to the growth of total output? (Consider both the effect of zero population growth on the steady-state growth path, and the transition from the "old" positive population growth to the "new" zero population growth steady-state growth path.)

6. Suppose somebody who hasn't taken any economics courses were to ask you why humanity escaped from the Malthusian trap--of very low standards of living and slow population growth rates that nevertheless put pressure on available natural resources and kept output per worker from rising--in which humanity found itself between the year 8000 B.C.E. and 1800. What answer would you give? (One paragraph only, please!)

7. At the end of the 1990s it appeared that because of the computer revolution the rate of growth of the efficiency of labor in the United States had doubled, from 1 percent per year to 2 percent per year. Suppose this increase were to be permanent. And suppose the rate of labor force growth were to remain constant at 1 percent per year, the depreciation rate were to remain constant at 3 percent per year, and the American savings rate (plus foreign capital invested in America) were to remain constant at 20 percent per year. Assume that the efficiency of labor in the U.S. in 2000 is $15,000 per year, and that the diminishing-returns-to-capital parameter α is 1/3.

a. What is the change in the steady-state capital-output ratio? What is the new capital-output ratio?

b. How would such a permanent acceleration in the rate of growth of the efficiency of labor change your forecast of the level of output per worker in 2040?

8. Suppose somebody who hasn't taken any economics courses were to ask you why it is that some countries are so very, very much poorer than others in the world today. What answer would you give? (Two paragraphs only, please!)