Economics 101b; Fall 2001; Problem Set 3

Due in class September 24, 2001

1. Suppose that the economy is well-described by the Solow growth model, with the diminishing-returns-to-capital parameter α= 1/3, the depreciation rate δ = .04 per year, the population growth rate n=.02 per year, and the rate of increase of the efficiency of labor g=.02 per year.

a. Suppose that the national savings rate s = 0.32, 32%. What is the steady-state capital output ratio?

b. Suppose that increased investment incentives and a large government budget surplus lower the savings rate s to 24%. What is the new steady-state capital output ratio?

c. Suppose that in the year 2000 the efficiency of labor E is $10,000 a year. What is the level of GDP per worker in 2000 if the economy is on the steady-state growth path given in (a)? What is the level of GDP per worker in 2030 if the economy remains on the steady-state growth path given in (a)?

d. How much, in percentage terms, is the steady-state growth path given in (b) below the steady-state growth path given in (a)?

e. At approximately what rate will the capital output ratio converge towards its new steady state value in the year in which the savings rate is changed?

a. Four

b. Three

c. With a steady-state capital-output ratio of 4, and a diminishing returns to scale parameter of 0ne-third, output-per-worker in 2000 is twice the efficiency of labor: $20,000. In 2030, with 2% per year growth in the efficiency of labor, output-per worker will be $36,227.

d. The capital-output ratio is 3/4, (3/4)^(1/2) is 0.866. So the second path is 13.4% below the first.

e. $(1-\alpha)(n+g+\delta) = .0533$. So the economy closes 5.33% of the gap to its new steady state each year.

2. Suppose that the economy is well-described by the Solow growth model, with the diminishing-returns-to-capital parameter $\alpha = 1/4$, the depreciation rate $\delta = .04$, the population growth rate n=.02, and the rate of increase of the efficiency of labor g=.02.

a. Suppose that the national savings rate s = 0.32, 32%. What is the steady-state capital-output ratio? If the efficiency of labor in year 2000 is equal to $20,000 a year, what is the steady-state growth path level of output per worker in 2000? Knowing that total consumption C is equal to (1-s) x Y—that consumption is the amount of national product not devoted to savings and investment—what then is the steady-state growth path level of consumption per worker in 2000?

b. Suppose that the national savings rate s = 0.48, 48%. What is the steady-state capital-output ratio? If the efficiency of labor in year 2000 is equal to $20,000 a year, what is the steady-state growth path level of output per worker in 2000? What is the steady-state growth path level of consumption per worker in 2000?

c. Can you explain why your level of consumption per worker in case (b) was different from your level of consumption per worker in case (a)? Was it lower or higher?

a. The steady-state capital-output ratio is 4. Steady-state growth path output per worker is $31,750. Steady-state consumption per worker is $21,590.

b. The steady-state capital-output ratio is 6. Steady-state growth path output per worker is $36,340. Steady-state consumption per worker is $18,900.

c. We would expect it to be different—the economy has a different savings rate and thus a different steady-state growth path, after all. The interesting thing about this example is that more savings and more capital leads to higher capital intensity, higher output per worker, but lower steady-state consumption per worker. The extra output produced by the extra capital is more than offset by the higher investment effort needed to maintain the higher capital intensity. Everyone would be better off if savings were reduced. The current generation would live much better—they could consume more. Future generations would have higher consumption per capita as well.

3. Suppose that population growth depends on the level of output per worker, so that:

(1) n = (.0001) x [(Y/L) - $200]

the population growth rate n is zero if output per worker equals $200, and that each $100 increase in output per worker raises the population growth rate by 1% per year.

Suppose also that the economy is in its *Malthusian* regime, so that the rate of increase of the efficiency of labor E is zero. Thus output per worker is given by:

(2) $$\frac{Y_t}{L_t} = \left(\frac{s}{n+\delta}\right)^{\left(\frac{\alpha}{1-\alpha}\right)} E_0$$

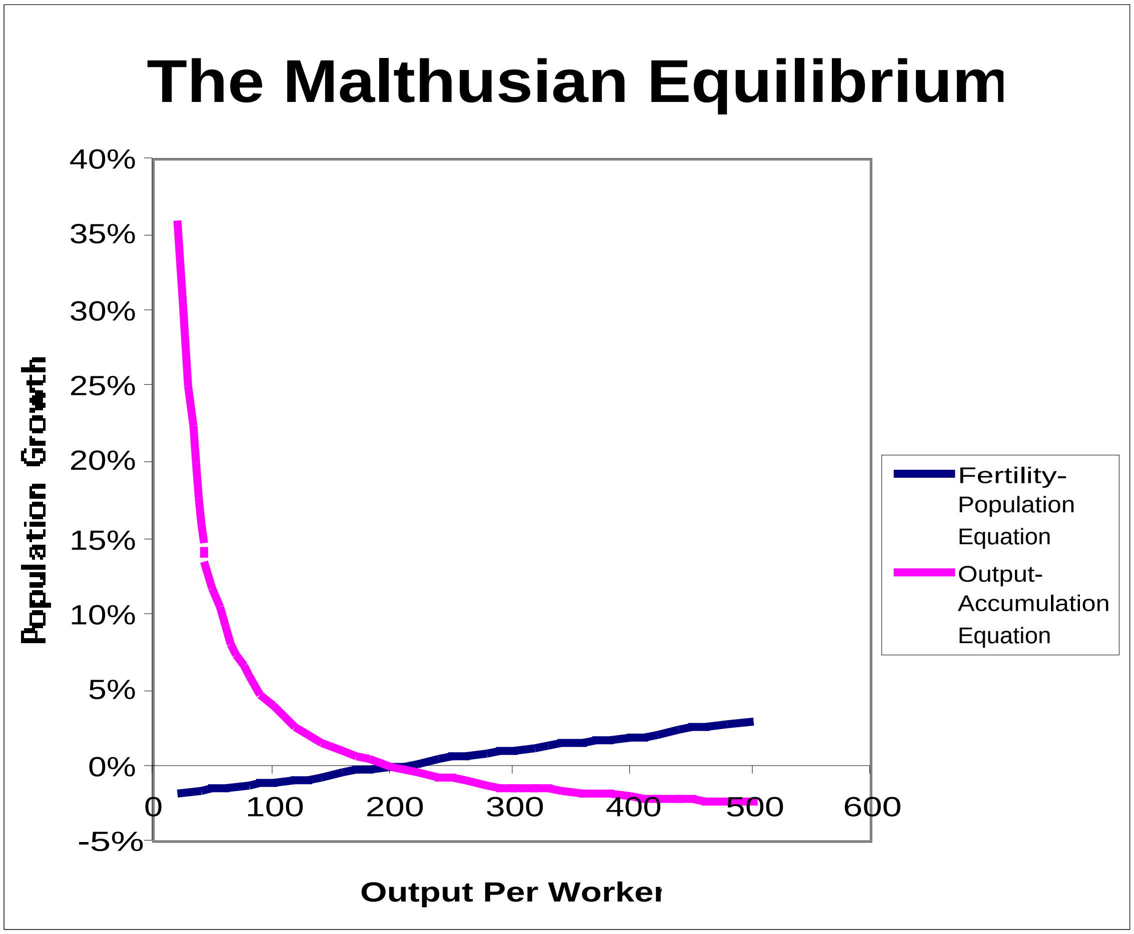
with the diminishing-returns-to-investment parameter α = .5, with the depreciation rate δ = .04, and with the efficiency of labor E_0 = $100.

a. Suppose that the savings rate s is equal to .08, 8% per year. Graph (on the same set of axes) steady-state output-per-worker (Y/L) as a function of the population growth rate n from equation (2) and the population growth rate n as a function of output-per-worker (Y/L) from equation (1).

b. Where do the curves cross? For what levels of output per worker Y/L and population growth n is the economy (i) on its steady-state path, and (ii) at its Malthusian rate of population growth?

c. Suppose that the savings rate were to rise by an infinitesimal amount--say by one-hundredth of one percentage point, from .08 to .0801. Calculate approximately how the equilibrium position of the economy would change. By how much--and in which direction--would steady-state output per worker change? By how much--and in which direction--would the population growth rate change?

a. Answer:



b. At an output per worker level of $200, population growth is zero. At a population growth rate of zero, steady-state output per worker is $200.

c. At an output per worker level of $200.167 the economy is in Malthusian equilibrium with a population growth rate of .00167% per year

4. Suppose that the economy is well described by the Solow growth model, with the diminishing-returns-to-investment parameter α= 1/2, the depreciation rate δ = .03, the population growth rate n=.01, and the rate of increase of the efficiency of labor g=.01. Suppose that the savings rate s = 20 and that in year 2000 the efficiency of labor E is $10,000.

a. What is the steady-state capital-output ratio?

b. What is the steady-state level of output per worker Y/L in 2000?

c. Suppose that actual output per worker in 2000 is $35,000. Is output per worker above or below its steady-state value?

d. Suppose that actual output per worker in 2000 is $35,000. Using the approximation that each year the economy closes a fraction:

$$(1-\alpha) \times (n + g + \delta)$$

of the gap between its current level of output per worker and its steady-state value of output per worker, calculate (approximately) what output per worker will be in 2001.

e. Suppose that actual output per worker in 2000 is $35,000. Using the same approximation, what (approximately) will output per worker be in 2010?

a. Four

b. $40,000

c. Below its steady-state value—output per worker should be growing relatively rapidly

d. Steady-state output per worker grows at 1%, and the economy closes 2.5% of the gap between current and steady-state output per worker each year. The gap is equal to 14% of output per worker. So output per worker in 2001 will be approximately (1 + 1% + 2.5% x 15%) x $35,000
= 1.01375 x 35000 = $35,481

e. An economy with output per worker growing at 1.375% per year starting from an initial output per worker level of $35,000 will reach $40,160 after ten years. An alternative approximation that takes account of the fact that growth slows as the economy reduces the gap to the steady-state growth path would produce an estimate that would be somewhat lower...

5. Many project that by the middle of the twenty-first century the population of the United States will be stable. Using the Solow growth model, what would such a downward shift in the growth rate of the labor force do to the growth of output per worker and to the growth of total output? (Consider both the effect of zero population growth on the steady-state growth path, and the transition from the "old" positive population growth to the "new" zero population growth steady-state growth path.)

A move to zero population growth would raise the steady-state capital output ratio and raise the level of output per worker along the steady-state growth path. As long as the economy was in the transition to the new steady-state growth path, output per worker would be growing more rapidly than before. Once the steady-state growth path is reached, output per worker growth would be the same: along the steady-state growth path, output per worker growth is determined by technological progress.

The rate of growth of total output would, of course, slow down: no growth in the number of workers means slower growth in total output.

6. Suppose somebody who hasn't taken any economics courses were to ask you why humanity escaped from the Malthusian trap--of very low standards of living and slow population growth rates that nevertheless put pressure on available natural resources and kept output per worker from rising--in which humanity found itself between the year 8000 B.C.E. and 1800. What answer would you give? (One paragraph only, please!)

My answer: First, the demographic transition: once incomes rose high enough, population growth rates slowed down—and slower population growth means higher steady-state capital output ratios and a richer world. What made incomes rise high enough for the demographic transition to begin? An acceleration of technological progress caused by the long slow accumulation of technologies for growing food, manipulating matter, and transmitting information. And once you have passed critical technological mass, technological progress continues rapidly.

7. At the end of the 1990s it appeared that because of the computer revolution the rate of growth of the efficiency of labor in the United States had doubled, from 1 percent per year to 2 percent per year. Suppose this increase were to be permanent. And suppose the rate of labor force growth were to remain constant at 1 percent per year, the depreciation rate were to remain constant at 3 percent per year, and the American savings rate (plus foreign capital invested in America) were to remain constant at 20 percent per year. Assume that the efficiency of labor in the U.S. in 2000 is $15,000 per year, and that the diminishing-returns-to-capital parameter α is 1/3.

a. What is the change in the steady-state capital-output ratio? What is the new capital-output ratio?

b. How would such a permanent acceleration in the rate of growth of the efficiency of labor change your forecast of the level of output per worker in 2040?

a. The steady-state capital-output ratio would fall from 4 to 3.33. But when productivity growth changes, the actual capital-output ratio remains constant. It doesn't drop instantaneously to 3.33; it remains at 4, and then gradually over time drifts down to 3.33

b. This economy covers 4% of the gap between its current position and its steady state each year. This means it closes 80% of the gap in 40 years—a large enough fraction for us to be comfortable using the assumption that the economy has converged to its steady state growth path. Along the new steady-state growth path, in 2040 the efficiency of labor will be higher by 49% than along the old path, but the capital-output ratio will be 16.7% lower. Taking account of these two factors, forecast output per worker will be 24.2% higher.

8. Suppose somebody who hasn't taken any economics courses were to ask you why it is that some countries are so very, very much poorer than others in the world today. What answer would you give? (Two paragraphs only, please!)

Rich countries are rich today because they (a) are culturally close enough to the heartland of the industrial revolution to take advantage of productive industrial technologies, (b) have relatively uncorrupt governments that encourage savings and investment, and (c) have passed through the demographic transition and thus have low rates of labor force growth.

Poor countries are poor today because they have not been culturally close to the heartland of the industrial revolution, hence technology transfer has been slow and uneven, and have not had uncorrupt governments that encourage savings and investment. Because they are poor vicious circles have set in: the capital goods needed to boost investment and capital intensity are very expensive; the demographic transition has not set in because in places of high mortality and low productivity your ability to avoid starving to death in your old age depends on having enough children that some will survive to support you.