Problem Set 2, Economics 202, Fall 1998
The Ramsey Model

1. Growth with Natural Resources:

Suppose that we have a modified Solow model in which the production function is given by:

$$Y_t = K_t^{\alpha} R_t^{\beta} (A_t L_t)^{1-\alpha-\beta}$$

and that

dR/dt = 0, dA/dt=gA, dL/dt=nL, dK/dt = sY - δK

a. Let Z be the capital-output ratio: K/Y. Derive an expression for the growth rate of Z in terms of Z(t) and the exogenous parameters of the model.

b. What does the expression you derived tell us about the long-run growth path of the economy?

2. Forced Savings-Forced Consumption:

a. Suppose that, except for the determination of consumption, the economy is as described by the Solow and the Ramsey-Cass-Koopmans models. Suppose that consumption per unit of effective labor is given by:

$$c_t = \frac{C_t}{A_t L_t} = \begin{cases} \bar{c} & \text{if } f(k_t) \geq \bar{c} \\ f(k_t) & \text{if } f(k_t) \leq \bar{c} \end{cases}$$

I. Derive an expression for dk/dt as a function of k and c
II. Find an expression for dk/dt as a function of k and exogenous parameters.
III. How many steady-state values of k are there? Zero? One? Two? Are steady-state values (if any) stable? Which? What will be the behavior of k over time given its initial value.

3. Forced Savings-Forced Consumption II:

Now suppose that in the model of (2) initially c is determined by infinitely-lived households with the usual Ramsey constant-relative-risk-aversion preferences, so that initially the economy is a standard Ramsey-Cass-Koopmans economy. Assume this economy is on its balanced growth path.

Now suppose that at some time (which we will call t=0) the government unexpectedly imposes the forced saving-forced consumption consumption plan on the economy for a period of length T.

After period T, the economy reverts to standard Ramsey-Cass-Koopmans behavior.

Describe the behavior of the economy--k and c--from time 0 to time T, and after time T. Phase diagrams and qualitative answers are all that is expected (or desired).

4. The Productivity Slowdown in Ramsey-Cass-Koopmans:

Suppose a Ramsey-Cass-Koopmans economy is originally, at time t=0, on its steady-state growth path. Suppose that at time t=0 labor augmenting technological change stops: g falls suddenly from its previous, positive value to zero.

Describe--phase diagams and qualitative answers--how k and c evolve over time after this productivity slowdown begins. Report--again qualitatively--how the capital stock per worker K/L and consumption per worker C/L evolve after the productivity slowdown begins. Compare them to the levels of the capital stock and consumption per worker K/L and C/L in the absence of a productivity slowdown.

5. Depreciation:

How would the first-order conditions (for consumption per effective worker c, capital per effective worker k, and the co-state variable λ) in the Ramsey-Cass-Koopmans model be different if we added depreciation back into the model? How would the phase diagram be different? Once again, qualitative answers only.