Economics 202a, Fall 1998
Lecture 7:
Government in the Ramsey Model

Adding government to the model:
Assume that the government buys output at rate G per unit of effective labor. Assume that government purchases do not affect utility from private consumption. Assume that government purchases do not affect future output. Assume that government purchases are financed by lump-sum taxes--for now of amount G per unit of effective labor, balanced budget. (Note: we have to use big G to avoid confusion with little g, the growth rate of labor-augmenting technology).

Investment is now equal to production minus private consumption minus government purchases:

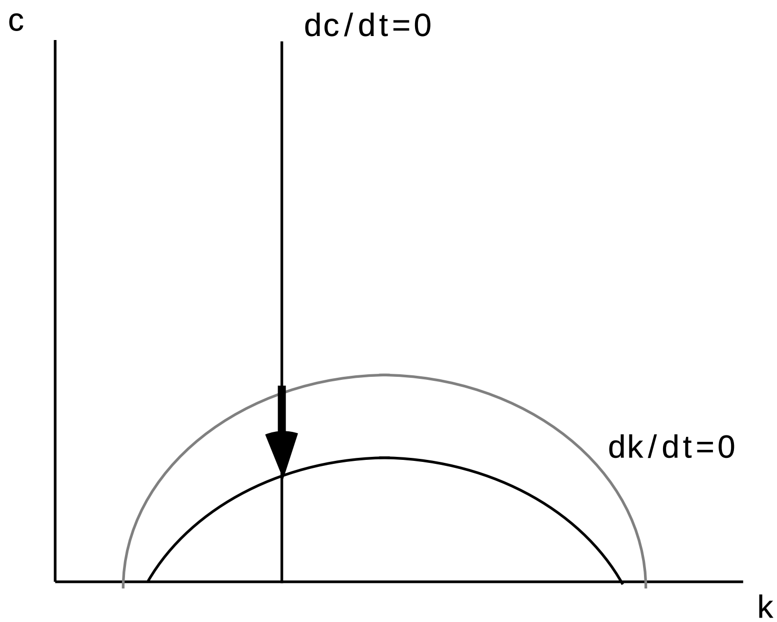
$$\frac{dk_t}{dt} = f(k_t) - c_t - G_t - (n+g)k_t$$

Taxes affect households' lifetime budget constraint--but they do not affect the Euler equation:

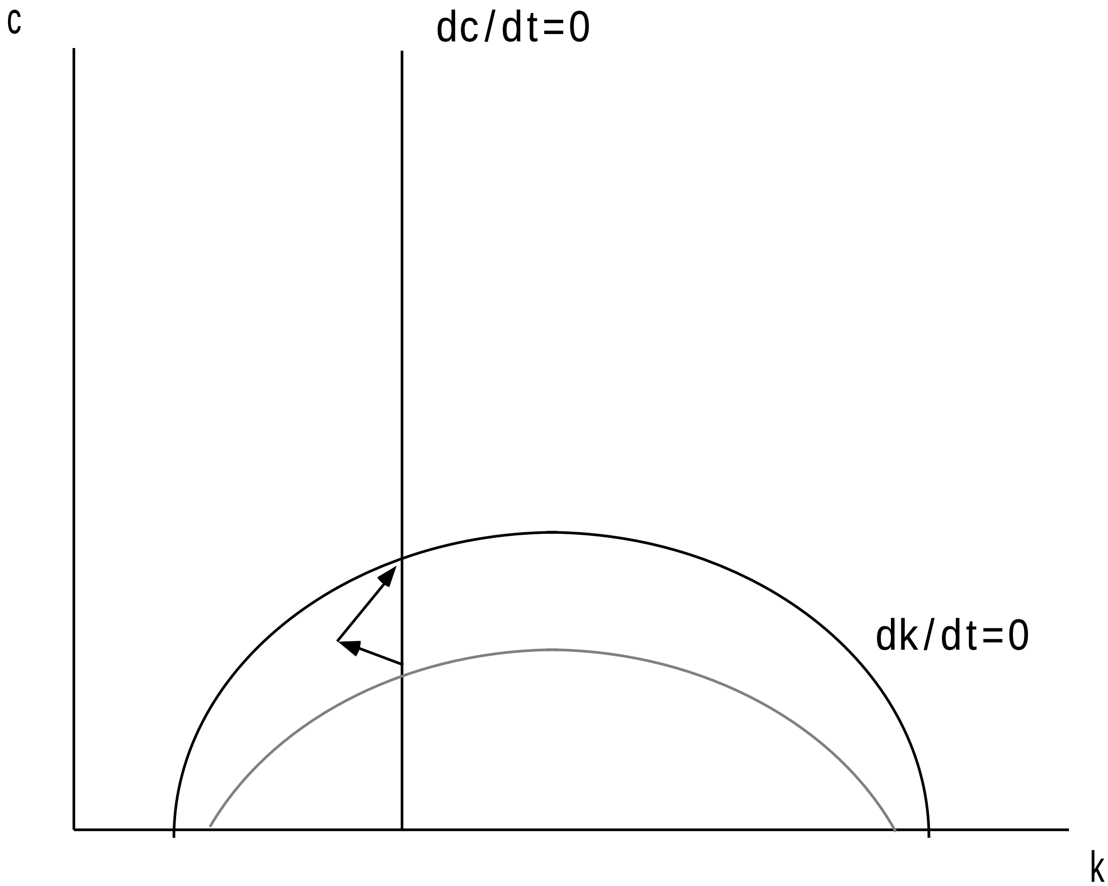
$$\frac{dc_t}{dt}\frac{1}{c_t} = \frac{f'(k_t) - (n+g) - \beta}{\theta} = \frac{f'(k_t) - \rho - \theta g}{\theta}$$

What's the consequence of the change in household's lifetime budget constraint? From PV of c = PV of future labor income plus V of initial capital stock to PV of c plus PV of G = PV of future labor income plus V of initial capital stock

Effect of a Permanent Change in Government Purchases:
Phase diagram:



Effect of a Temporary Change in Government Purchases:
Phase diagram:



Bond and Tax Finance:

The government's budget constraint:

$$\int_{t=0}^{\infty} e^{-R_t} G_t e^{(n+g)t} dt = -b_0 + \int_{t=0}^{\infty} e^{-R_t} T_t e^{(n+g)t} dt$$

$$\frac{db_t}{dt} = r_t b_t - (n+g) b_t + G_t - T_t$$

$$\lim_{s \to \infty} e^{-R_s} e^{(n+g)s} b_s = 0$$

The household's budget constraint:

$$\int_{t=0}^{\infty} e^{-R_t} e^{(n+g)t} c_t dt \le k_0 + b_0 + \int_{t=0}^{\infty} e^{-R_t} e^{(n+g)t} \left[w_t - T_t \right] dt$$

$$\int_{t=0}^{\infty} e^{-R_t} e^{(n+g)t} c_t dt \le k_0 + b_0 + \int_{t=0}^{\infty} e^{-R_t} e^{(n+g)t} \left[w_t \right] dt - \int_{t=0}^{\infty} e^{-R_t} e^{(n+g)t} \left[T_t \right] dt$$

$$\int_{t=0}^{\infty} e^{-R_t} e^{(n+g)t} c_t dt \le k_0 + \int_{t=0}^{\infty} e^{-R_t} e^{(n+g)t} \left[w_t \right] dt - \int_{t=0}^{\infty} e^{-R_t} e^{(n+g)t} \left[G_t \right] dt$$

Thus there is a sense in which *only the path* (indeed, only the present value--although note that government purchases can affect real interest rates) *of government purchases affects the economy*. Bond vs. taxes are *irrelevant* for understanding where the economy is going.

Ricardian Consumers vs. Keynesian Propensities

- Entrance of new households into the economy
- Bequests and "internal optima": strategic bequests; liquidity constraints; non-lump-sum taxes; rule-of-thumb consumption behavior.
- Is everything neutral?

On to the Diamond Overlapping-Generations Model.

Let's move from continuous to discrete time. Let's continue to let the labor force grow at n and labor-augmenting technology at rate g. Let's have everyone live for two periods--be young and old. The young save; the old dissave; the young work; the old don't work.

Constant relative risk aversion utility and simple budget constraint:

$$U_t = \frac{C_{1,t}^{1-\theta}}{1-\theta} + \frac{1}{1+\rho} \frac{C_{2,t+1}^{1-\theta}}{1-\theta}$$

$$C_{2,t+1} = (1+r_{t+1})(w_t A_t - C_{1,t})$$

$$C_{1,t} + \frac{1}{1+r_{t+1}} C_{2,t+1} = A_t w_t$$

Form and maximize the Lagrangian:

$$L = \frac{C_{1,t}^{1-\theta}}{1-\theta} + \frac{1}{1+\rho}\frac{C_{2,t+1}^{1-\theta}}{1-\theta} + \lambda_t \left[C_{1,t} + \frac{1}{1+r_{t+1}} C_{2,t+1} - A_t w_t \right]$$

$$C_{1,t}^{-\theta} = \lambda_t$$

$$\frac{1}{1+\rho} C_{2,t+1}^{-\theta} = \frac{\lambda_t}{1+r_{t+1}}$$

$$\frac{1}{1+\rho} C_{2,t+1}^{-\theta} = \frac{C_{1,t}^{-\theta}}{1+r_{t+1}}$$

$$\frac{C_{2,t+1}}{C_{1,t}} = \left[\frac{1+r_{t+1}}{1+\rho} \right]^{1/\theta}$$

Savings:

$$C_{1,t} + \frac{(1+r_{t+1})^{\frac{1-\theta}{\theta}}}{(1+\rho)^{\frac{1}{\theta}}} = A_t w_t$$

$$C_{1,t} = \frac{(1+\rho)^{1/\theta}}{(1+\rho)^{1/\theta} + (1+r_{t+1})^{(1-\theta)/\theta}} A_t w_t$$

$$s(r_{t+1}) = \frac{C_{1,t}}{A_t w_t} = \frac{(1+r_{t+1})^{(1-\theta)/\theta}}{(1+\rho)^{1/\theta} + (1+r_{t+1})^{(1-\theta)/\theta}}$$

$$C_{1,t} = [1 - s(r_{t+1})] A_t w_t$$

Are savings an increasing function of the interest rate? Income and substitution effects opposed. s(r) is increasing in r if and only if theta is less than one. With log utility the two effects balance.

$$K_{t+1} = s(r_{t+1}) L_t A_t w_t$$

$$k_{t+1} = \frac{s(r_{t+1}) w_t}{(1+n)(1+g)}$$

$$k_{t+1} = \frac{s(f'(k_{t+1}))[f(k_t) - k_t f'(k_t)]}{(1+n)(1+g)}$$

Now let’s consider the special case of Cobb-Douglas production and log utility (theta=1):

$$s(r_{t+1}) = \frac{1}{2+\rho}$$

$$f(k_t) = k_t^{\alpha}$$

$$w_t = (1-\alpha)k_t^{\alpha}$$

$$k_{t+1} = \left(\frac{1}{(1+n)(1+g)}\right)\left(\frac{1}{2+\rho}\right)(1-\alpha)k_t^{\alpha} = Dk_t^{\alpha}$$

$$k^* = \left[\left(\frac{1}{(1+n)(1+g)}\right)\left(\frac{1}{2+\rho}\right)(1-\alpha)\right]^{\frac{1}{1-\alpha}} = D^{\frac{1}{1-\alpha}}$$

Linearize the equation of motion around k*:

$$\left.\frac{dk_{t+1}}{dk_t}\right|_{k_t=k^*} = D\alpha k^{*(1-\alpha)}$$

$$\left.\frac{dk_{t+1}}{dk_t}\right|_{k_t=k^*} = \alpha$$

$$k_{t+1} \approx k^* + \alpha(k_t - k^*)$$

$$k_t - k^* \approx \alpha^t(k_0 - k^*)$$

Note that in this model a “period” is half a person’s lifetime.
Note, also, that the *net* savings rate is the sum of the saving of the young and the dissaving of the old--hence is a decreasing function of k.
Note, third, that s(r) is bad notation: s in the Solow model was always savings gross of depreciation; s here is total demand for capital stock.

The General Case:

k(t+1) = (output per unit of effective labor) x (wage share) x (savings rate of young) / (1+growth of effective labor force).

Lots of things can happen...

Dynamic Inefficiency

Note that with k* as defined above, with g=0, utility logarithmic, and production Cobb-Douglas, then the marginal product of capital on the balanced growth path is:

$$f'(k^*) = \frac{\alpha}{1-\alpha}(1+n)(2+\rho)$$

The golden-rule capital stock is defined by:

$$f'(k_{GR}) = n$$

For alpha sufficiently small, the capital stock on the balanced growth path exceeds the Golden rule level.

What do we mean by “dynamic inefficiency”? Let’s introduce an infinitely-lived benevolent social planner into the Diamond model when alpha is sufficiently small...

Equilibrium of Diamond model can be Pareto inefficient--because there are an infinite number of agents. Stop the Diamond model at some period T--have young who eat up their wages in that period--and the reduction in the capital stock is not Pareto optimal. (If T is large, however, it is very hard to justify...)

In the market economy, holding capital is the only way to consume when you are old. The social planner does not have to make consumption of the old a function of the economy’s capital stock.

Government in the Diamond Model

Tax finance only:

$$k_{t+1} = \frac{1}{(1+n)(1+g)} \frac{1}{2+\rho} [(1-\alpha)k_t^{\alpha} - G_t]$$

Bond finance: savings of the young can take the form of bonds as well as capital:

$$k_{t+1} = \frac{1}{(1+n)(1+g)} \frac{1}{2+\rho} [(1-\alpha)k_t^{\alpha} - T_t] - b_{t+1}$$

Suppose k(t+1) were going to be above the golden rule value--the government could issue bonds instead of levying taxes, and so reduce the capital stock...