

## Econ 202b Midterm Exam, Fall 1998

(Do three out of four)

**1. Investment.** Consider the infinite-horizon q-theory model of investment with adjustment costs in which the flow of profits to a representative firm is given by:

$$\pi_t = (\alpha - \beta K_t/2)K_t$$

where  $K_t$  is the representative firm's capital stock. Suppose further that the firm maximizes the present value of profits for a constant discount rate  $r$ , and that the capital-stock adjustment costs of the representative firm are:

$$\gamma (I_t^2/2)$$

for some constant  $\gamma$ . Thus the firm's objective function is:

$$= \int_{t=0}^{\infty} e^{-rt} (\pi_t - I_t - \frac{\gamma I_t^2}{2}) dt$$

- a. Derive an equation for the locus of points along which  $q$ , the shadow value of a marginal unit of new capital, is constant.
- b. Where is the steady state of the system? Toward what point  $(K, q)$  does the system converge?
- c. Describe—in words and phase diagrams only—the reactions of  $q$  and  $K$  over time to the following shocks:
  - i. The government suddenly and unexpectedly imposes a *permanent* investment tax credit of amount  $\phi$ .
  - ii. The government suddenly and unexpectedly imposes a *temporary* corporate income tax at rate  $\tau$ .
  - iii. The government today announces that the interest rate  $r$  will be lowered next year, will remain at its (temporarily) low level for five years, and then return to normal.

**2. Search and Employment.** Suppose that there are a large number of firms in the economy that offer wages uniformly distributed between two levels: **L** (for lower bound) and **U** (for upper bound). In order to find out the wage associated with one particular open job, a worker must pay a cost **S** (for search).

After learning about the wage level, the worker can either (i) take the job at the offered wage, or (ii) look for another job (by paying an additional cost **S**).

The worker accepts or rejects the job in order to maximize:

$$E(w - nS)$$

The expected value **E** of the wage **w** he or she finally receives, minus the cost of search **S** times the number of jobs **n** searched.

- a. Suppose a worker has started searching for jobs. For what values of its wage **w** will the worker accept a newly sampled job?
- b. Are there circumstances under which the worker will always accept the first job offer, no matter where in [**L**, **U**] its wage is located? If so, what are they?

**3. Wage Bargaining.** Suppose that workers are represented by a union with the objective function:

$$[U(\mathbf{w}) - \mathbf{D}]L + U(\mathbf{w}_u)(N-L)$$

Where  $U$  is some increasing function of the income of the relevant group of workers,  $\mathbf{w}$  is the wage received by those hired by the representative firm,  $\mathbf{w}_u$  is the unemployment benefit paid to those not hired by the representative firm,  $\mathbf{D}$  is the disutility of work,  $L$  is the level of employment, and  $N$  is the level of the labor force. Assume that workers like to be employed—that the wage they receive minus the disutility of work is higher than the unemployment benefit.

Suppose that the representative firm's profits are:

$$A L^\alpha / \alpha - \mathbf{w}L, A > 0, 0 < \alpha < 1$$

Where  $A$  is stochastic, and  $\alpha$  is known and fixed. The union sets the wage  $\mathbf{w}$  after  $A$  is known, and the firm then chooses  $L$  to maximize its profits given  $\mathbf{w}$  and  $A$ .

- a. What is the firm's optimal choice of  $L$  as a function of  $\mathbf{w}$  and  $A$ ?
- b. What is the union's optimal choice of  $\mathbf{w}$  given  $A$ , and given the firm's reaction function in part (a) above?
- c. Suppose that the union gets to choose *both*  $\mathbf{w}$  and  $L$ —subject to the constraint that firm profits be at least some level  $\pi_0$ . Does this more powerful union always set wages to a higher level than in part (b) above? Why or why not?

**4. Precautionary Savings.** Suppose that we have a representative consumer with a time-separable constant relative risk aversion utility, an opportunity to invest funds at a constant net rate of return  $r$ , and a constant rate of time discount  $\rho$ .

Then consumption satisfies the Euler equation (first order condition):

$$C_t^{-\theta} = \frac{1}{1 + \rho} E_t[(1 + r)C_{t+1}^{-\theta}]$$

a. Suppose that  $r = \rho$ . Will consumption then follow a random walk with zero drift? Why or why not?

b. Suppose that  $r < \rho$ . Will consumption then follow a random walk with constant drift? Why or why not?

c. Suppose that the consumer in period  $t$  receives news that consumption in period  $t+1$  is going to be much more volatile than previously expected because a lot of news about permanent income will arrive in period  $t+1$ . On receiving the news that volatility will be high because news about permanent income will shortly be arriving, does he/she raise or lower period- $t$  consumption? Why?

d. Derive an approximate expression for the expected growth rate of consumption between period  $t$  and  $t+1$ , keeping terms that are first order in the variance of period  $t+1$  consumption.