Suggested Problem Set 1 Solutions

Economics 202B – Second Half

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9.3 Discrete-time IS-LM model with one period price stickiness
Setup:

$$\begin{aligned} y_t &= c - ar_t, \text{ IS curve} \\ m_t - p_t &= b + hy_t - ki_t, \text{ LM curve} \\ m_t &= m_{t-1} + u_t, \ E_{t-1}(u_t) = 0, \text{ money supply process} \\ i_t &= r_t + \pi_t^e \equiv r_t + E_t(p_{t+1} - p_t), \text{ Fisher identity} \end{aligned}$$

(a) First, note that by the law of iterated of expectations, $E_t(E_{t+1}(p_{t+2}) - p_{t+1}) = E_t(p_{t+2} - p_{t+1})$. Next, a shock at $t+1$ will be fully reflected in the price level at $t+2$ since prices are sticky for only one period. To see this consider the following situation. Say at time t the real money supply (in logs) is $\overline{m-p}$, then a shock hits that raises the money supply to $m_t + u_t$. If the price is constant for one period, the real money supply will have increased. Now, agents expect average shocks to be zero, so they expect the economy to return to the pre-shock equilibrium. Therefore, they expect the price to adjust at $t+1$ so that the real money supply equals $\overline{m-p}$ once again. Therefore, the price must increase by u_t at $t+1$. This is true at each period, therefore expected inflation at any time s equals the shock, that is

$$\pi_s^e \equiv E_s(p_{s+1} - p_s) = u_s, \forall\ s.$$

The shock at $t+1$, u_{t+1}, has expectation 0 at t, therefore the expected price change between $t+1$ and $t+2$ is 0, i.e.,

$$E_t(p_{t+2} - p_{t+1}) = 0. \tag{1}$$

To solve for the second part of the question, take the LM equation at $t+1$ and substitute the Fisher identity to obtain

$$m_{t+1} - p_{t+1} = b + hy_{t+1} - kr_{t+1} - kE_{t+1}[p_{t+2} - p_{t+1}]. \tag{2}$$

Next, take expectations of (2) and the results from (1) to arrive at

$$E_t(m_{t+1}) - E_t(p_{t+1}) = b + h\bar{y} - k\bar{r}, \tag{3}$$

where we have used the fact that output and the real interest rate deviate from their long-run values, $\bar{y}$ and $\bar{r}$, by the shock u_{t+1} and the expectation of this shock at t is zero.

(b) This part is nothing but algebra, so the solution will be as concise as possible. First, to solve for p_t re-arrange (3), substitute the money supply process into the expectation operator and subtract p_t from both sides to arrive at

$$E_t(p_{t+1} - p_t) = (m_t - p_t) - b + h\bar{y} + k\bar{r}. \tag{4}$$

Next, note that the left-hand side of (4) is simply expected inflation, which equals u_t. Using this fact and the money supply process for m_t, we can re-arrange (4) and solve for p_t:

$$p_t = m_{t-1} - b - h\bar{y} + k\bar{r}. \tag{5}$$

To solve for y_t, first solve for i_t from the LM curve and substitute for the $(m_t - p_t)$ term in this equation using (4) to give

$$i_t = \frac{h(y_t - \bar{y}) + k\bar{r} - u_t}{k}. \tag{6}$$

Next, substitute (6) into the IS curve, use the Fisher identity and remember that $\pi_t^e = u_t$ to arrive at

$$y_t = c - a\left[\frac{h(y_t - \bar{y}) + k\bar{r} - u_t}{k}\right] + au_t,$$

which re-arranging for y_t gives

$$y_t = \frac{kc + a[h\bar{y} - k\bar{r} + (1+k)u_t]}{k + ah}. \tag{7}$$

To determine the nominal interest rate simply substitute the solution for y_t into (6) to obtain

$$i_t = \frac{h(c - \bar{y}) + k\bar{r}}{k + ah} + \frac{ah - 1}{k + ah}u_t. \tag{8}$$

Finally, to solve for the real interest rate substituting (8) into the Fisher identity and remembering that $\pi_t^e = u_t$ to arrive at

$$r_t = \frac{h(c - \bar{y}) + k\bar{r} - (1+k)u_t}{k + ah}. \tag{9}$$

(c) To check whether the Fisher effect holds in this economy simply substitute π_t^e for u_t in (8) giving

$$i_t = \frac{h(c - \bar{y}) + k\bar{r}}{k + ah} + \frac{ah - 1}{k + ah}\pi_t^e, \tag{8'}$$

which implies that the change in expected inflation is not reflected one-for-one in the nominal interest rate. This occurs because prices are sticky for one period, which causes output and the nominal interest rate to adjust to clear the money market. In order to change output, the real interest rate must change, which in turn implies that the nominal interest rate need not adjust one-for-one with expected inflation.

9.5 Policy rules, rational expectations, and regime changes. (See Lucas, 1976, and Sargent, 1983.)
Setup:

$$\begin{aligned} y_t &= \bar{y} + b\left(\pi - \pi^e\right), \text{ Lucas supply curve} \\ m_t &= m_{t-1} + a + \varepsilon_t, \text{money supply, where } \varepsilon_t \text{ is a white noise disturbance} \\ y_t &= m_t - p_t, \text{ aggregate demand} \end{aligned}$$

(a) For the following note that $E(x_t|\Omega_{t-1}) \equiv E_{t-1}(x_t) \equiv x_t^e$.

$$\begin{aligned} \pi_t - \pi_t^e &= p_t - p_{t-1} - (p_t^e - p_{t-1}) = p_t - p_t^e, \text{ so} \\ m_t - p_t &= \bar{y} + b\left(\pi_t - \pi_t^e\right) = \bar{y} + b\left(p_t - p_t^e\right), \text{ which implies} \\ p_t &= \frac{m_t - \bar{y}}{1+b} + \frac{b}{1+b} p_t^e. \end{aligned} \tag{1}$$

Now assuming rational expectations we can solve for p_t^e by taking the expectation of equation (1) and re-arranging terms:

$$\begin{aligned} p_t^e &= m_t^e - \bar{y} \\ &= E_{t-1}\left(m_{t-1} + a + \varepsilon_t\right) - \bar{y} \\ &= m_{t-1} + a - \bar{y}. \end{aligned}$$

Finally, to solve for y_t we must solve for $p_t - p_t^e$:

$$\begin{aligned} p_t - p_t^e &= \frac{m_t - \bar{y}}{1+b} + \frac{b}{1+b} p_t^e - p_t^e \\ &= \frac{m_t - \bar{y} - p_t^e}{1+b} \\ &= \frac{m_t - m_{t-1} - a}{1+b}, \text{ so using the supply curve} \\ y_t &= \bar{y} + \frac{b}{1+b}\left(m_t - m_{t-1} - a\right). \end{aligned} \tag{2}$$

(b) One can see from equation (2) that we need to know m_t, m_{t-1} and a in order to determine current output. The intuition behind this result, is that since only *unexpected money* (ε_t) affects output, we need to know a to determine how much of the change in money supply between period t and

$t+1$ was due to the unexpected shock.

(c) $m_t = m_{t-1} + \varepsilon_t$ with probability ρ, so expected money is now:

$$\begin{aligned} m_t^e &= \rho\,(m_{t-1}) + (1-\rho)\,(m_t - m_{t-1} + a) \\ &= m_{t-1} + (1-\rho)a. \end{aligned}$$

Therefore, going through similar steps as in part (a) one can show the following:

$$\begin{aligned} p_t^e &= m_t^e - \bar{y} = m_{t-1} + (1-\rho)a - \bar{y}, \\ p_t - p_t^e &= \frac{m_t - \bar{y} - p_t^e}{1+b} = \frac{m_t - m_{t-1} - (1-\rho)a}{1+b}, \\ y_t &= \bar{y} + \frac{b}{1+b}\left[m_t - m_{t-1} - (1-\rho)a\right]. \end{aligned} \tag{3}$$

(d) First, note that equation (2) holds for any time period if there is no regime shift, so we can write:

$$y_{t-1} = \bar{y} + \frac{b}{1+b}(m_{t-1} - m_{t-2} - a). \tag{4}$$

Next subtract (4) from (2) to obtain

$$\Delta y_t \equiv y_t - y_{t-1} = \frac{b}{1+b}(\Delta m_t - \Delta m_{t-1}), \tag{5}$$

which implies that if there are no regime shifts output growth is determined by the change in money growth. Now, if there is a regime shift at t the change in output can be found by subtracting (4) from (3) to arrive at

$$\Delta y_t = \frac{\rho a b}{1+b} + \frac{b}{1+b}(\Delta m_t - \Delta m_{t-1}). \tag{6}$$

The null hypothesis that the central bank's announcement of a regime shift has no credibility can be tested by testing whether $\rho = 0$ in (6). Therefore, if the announcement is not believed equations (5) and (6) are identical. One can estimate ρ as follows. Regress Δy_t on $(\Delta m_t - \Delta m_{t-1})$ and a dummy variable that equals one in the period there is a regime shift. The coefficient on the dummy will reflect the credibility of the central bank's announcement. Since a and b are known, we can then determine ρ given the coefficient on the dummy variable.

9.12 The tradeoff between low average inflation and flexibility in response to shocks with delegation of control over monetary policy. (Rogoff, 1985.)
Setup:

$$y = \bar{y} + b\left(\pi - \pi^e\right), \text{ Lucas supply curve}$$
$$SWF = \gamma y - \frac{a\pi^2}{2}$$
$$E(\gamma) = \bar{\gamma}, Var(\gamma) = \sigma_\gamma^2$$
$$CB = c\gamma y - \frac{a\pi^2}{2}$$

(a) The policy maker will maximize the CB function by choosing the inflation rate subject to the Lucas supply curve:

$$\max_\pi CB = c\gamma y - \frac{a\pi^2}{2}$$
$$\text{s.t. } y = \bar{y} + b(\pi - \pi^e), \text{ therefore}$$
$$\max_\pi c\gamma[\bar{y} + b\left(\pi - \pi^e\right)] - \frac{a\pi^2}{2}.$$

The resulting first-order condition is:

$$\frac{\partial CB}{\partial \pi} = c\gamma b - a\pi = 0, \text{ which implies}$$
$$\pi^* = \frac{cb\gamma}{a}.$$

Therefore, a central banker who cares more about inflation will chose a lower c.

(b) By rational expectations $\pi^e = E(\pi)$, therefore

$$\pi^e = E\left(\frac{cb\gamma}{a}\right) = \frac{cb}{a}E(\gamma) = \frac{cb\bar{\gamma}}{a}.$$

(c) Simply substitute for π and π^e from parts (a) and (b) into the SWF function and take the expectation:

$$\begin{aligned} E(SWF) &= E\left(\gamma y - \frac{a\pi^2}{2}\right) \\ &= E\left\{\gamma\left[\bar{y} + b\left(\frac{cb\gamma}{a} - \frac{cb\bar{\gamma}}{a}\right)\right]\right\} - \frac{a}{2}E\left(\frac{c^2b^2\gamma^2}{a^2}\right) \\ &= \bar{\gamma}\bar{y} + \frac{cb^2}{a}E\left(\gamma^2\right) - \frac{cb^2}{a}\bar{\gamma}E\left(\gamma\right) - \frac{c^2b^2}{2a}E\left(\gamma^2\right) \\ &= \bar{\gamma}\bar{y} + \frac{cb^2}{a}\sigma_\gamma^2 - \frac{c^2b^2}{2a}\left(\bar{\gamma}^2 + \sigma_\gamma^2\right). \end{aligned}$$

(d) Now simply maximize the solution to part (c) subject to the policy variable c:

$$\frac{\partial E(SWF)}{\partial c} = \frac{b^2}{a}\sigma_\gamma^2 - \frac{cb^2}{a}\left(\bar{\gamma}^2 + \sigma_\gamma^2\right) = 0, \text{ which implies}$$
$$c^* = \frac{\sigma_\gamma^2}{\sigma_\gamma^2 + \bar{\gamma}^2}.$$

There is a trade-off here. In solving for the optimal inflation rate chosen by the central banker in (a), we can see that choosing a more conservative banker, i.e., one with low c, produces a better performance in terms of average inflation. However, such a central banker would not respond well to the shocks. Thus there is some optimal level of conservatism that balances these two forces.

The value of c^* that maximizes the expected value of true social welfare is decreasing in $\bar{\gamma}$. Since we know that π^e will equal π on average (since γ will equal $\bar{\gamma}$ on average), output will equal full-employment output on average, regardless of the values of c or $\bar{\gamma}$. From part (a), we can see that if γ is higher on average, inflation will also be higher on average, for a given c. Therefore, it will be welfare improving to offset this and keep inflation lower on average by having a central banker with a lower c.

The value of c^* is increasing in σ_γ^2. Therefore, the more variable are the shocks, the less conservative the central banker should be. Since the central banker can act after γ is realized, she can choose to offset any deviation in γ from its expected value, which will raise welfare. The central banker will do this only to the extent that she cares about the shock's effect. Therefore. the more that γ varies, the better it is to have a central banker who cares about the shock's effect and will act to offset it.